Journal of Engineering Physics and Thermophysics, Vol. 79, No. 2, 2006

CARRYING CAPACITY OF A CYLINDRICAL MAGNETOFLUID SUPPORT

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UDC 537.84

The influence of magnetic and geometric parameters on the carrying capacity of a magnetofluid support consisting of a cylindrical bushing and a journal suspended in a linearly magnetizable fluid has been investigated. The existence of the absolute maximum of a hydrostatic buoyancy force for the optimum relation of the bushing and journal radii has been found. Simple interpolation formulas which describe the dependence of the characteristics of the maximum on the susceptibility of the fluid with an accuracy sufficient for technical calculations have been obtained.

Introduction. Forces of magnetic origin that are comparable to the gravity force and may significantly exceed it in certain cases act on bodies suspended in a magnetic fluid in the presence of a magnetic field. This effect finds application in hydrostatic supports (a solid body is stably suspended inside the volume of a fluid with a lower density), separators (separating an ore for the density), and other devices [1–4]. Magnetic hydrostatic forces depend in a complicated manner on the geometric and magnetic characteristics of bodies. The range of problems for which the analytical dependences have been obtained is very limited [5–8]. In particular, the problem is easily solved if the dimensions of a body are smaller than the characteristic dimension of the inhomogeneity of a field (i.e., than the dimensions of the field source) and if the disturbances introduced by a small body are insignificant as compared to a magnetizing field. In this case, the magnetic force, just as the Archimedes force, depends only on the volume of the body, not on its shape, and its value and direction are determined by the gradient of the magnetizing field. This approximation solves, in the main, the problem of calculation of magnetofluid separators in which the magnetic force acts on the particles of a crushed material. The force-to-volume ratio (which may exceed hundreds of times the density of gravity forces), not the force itself (which is insignificant because of the smallness of the particles), is of crucial importance for these devices.

The carrying capacity of a magnetofluid support is determined by the absolute value of the force. In this case, the body is not small in the sense indicated above, and its volume is comparable to the volume in which the field is produced; consequently, not only does the magnetic buoyancy force depend on the gradient of the field and the volume of the suspended body but it also depends on its geometric features. The regularities of these dependences have not been adequately developed at present. The results of investigation of spherical bodies in a magnetic fluid in the approximation of a low magnetic susceptibility have been presented in [9]. An important result of this work was finding the existence of an optimum relation of the radii of a magnet and an external magnetic sphere, for which the force reaches its absolute maximum, which is important for designing magnetofluid supports, bearings, and suspensions. It would appear natural that such a maximum must exist in the case of cylindrical geometry, too. The solution of the cylindrical problem has been considered in [2, 6], but this property of the force has not been noted there.

The present work seeks to numerically investigate the extremum properties of the force with the use of the model of [6] and to construct simple interpolation formulas which will enable us to calculate the carrying capacity of a cylindrical magnetofluid support.

General Formulation of the Problem on Interaction of Magnets and Nonmagnetic Bodies in Magnetic Fluids. The equilibrium pressure distribution in magnetic fluids will be written as [1, 2]

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$$p = p_0 - \rho g (z - z_0) + \mu_0 \int_{H_0}^{H} M(H)\, dH . \quad (1)$$

The force acting on a body on the source side of a fluid is determined as the integral of forces distributed on the surface of the body S. For a nonmagnetic body in a magnetic fluid, this integral has the form [1, 2]

$$\mathbf{F} = - \oint_S \left[p + \frac{1}{2} \mu_0 (M\mathbf{n})^2 \right] \mathbf{n} dS , \quad (2)$$

where $\mathbf{n}$ is the vector of the external normal to the portion of the surface dS. The second term in the integrand is determined by the jump of Maxwell stresses in crossing the boundary of media with different magnetic properties.

In integrating over a closed surface, coordinate-independent components in the pressure distribution yield a resultant force equal to zero. In integrating, hydrostatic pressure due to the gravity force gives rise to the Archimedes force, which will be not considered subsequently for the sake of brevity. Thus, we assume that p in (2) represents only the magnetofluid pressure whose resultant action leads to an ejection of the body in the direction of decrease in the field. The force (2) is appropriately defined in this case as the magnetic buoyancy force.

Whereas nonmagnetic bodies are capable of floating under the action of the lifting force produced by the inhomogeneous external field, magnetic bodies are capable of self-levitating since they themselves are the sources of a magnetic field. A magnet placed in a vessel with a magnetic fluid tends to occupy a stable position under the action of the force that is also determined by expression (2).

To compute (2) we must know the equation of magnetization of a magnetic fluid. The Langevin dependence may be used in the general case. However, in solving the hydrostatic problem, it is more efficient to use the linear-fractional approximation [10], which enables us to obtain the analytical expression for the integral (1):

$$p = \mu_0 \int_{H_0}^{H} \frac{M_s H dH}{H_h + H} = p_h \left(\frac{H}{H_h} - \ln \left(1 + \frac{H}{H_h} \right) \right), \quad (3)$$

where M_s and H_h are the saturation magnetization and the halved-magnetization field dependent on the kind of fluid (the saturation magnetization may attain values of the order of 100 kA/m, whereas the values of the halved-magnetization field lie in a comparatively narrow range (5–15 kA/m)) and $p_h = \mu_0 M_s H_h$ is the characteristic pressure.

To obtain the field distribution as a function of the coordinates we must solve a system of magnetostatic equations. Thereafter we may compute the integral (2) determining the force as a function of the geometric and magnetic parameters.

Analytical Model of a Cylindrical Magnetofluid Support. For analysis of the influence of the magnetic susceptibility of the fluid on the extremum characteristics of interaction of bodies in a cylindrical support, we use the analytical solution of the problem [6] whose geometry is presented in Fig. 1.

A cylindrical journal of radius R_2 is in a cylindrical nonmagnetic bushing of radius R_1 filled with magnetizable fluid with a constant magnetic susceptibility χ. The journal and bushing axes are parallel to each other but are spaced r_0 apart. The journal is uniformly magnetized perpendicularly to the axis; its magnetization is equal to M_f. We assume that the length of the bushing and the journal is much larger than their diameters, so that the geometry of the problem is considered to be plane. For reasons of symmetry of the field distribution, the force acting on the journal will only have an x component, which is found by integration of the forces of magnetofluid pressure p and Maxwell stresses σ_n over the contour L enclosing the journal. In the case of the linear law of magnetization of the fluid $M = \chi H$, we may express the force acting on the magnet by the formulas

$$F_x = \oint_L (p\mathbf{n} + \sigma_n)\, \mathbf{i} dL = \mu_0 \mu l \oint_L \left(H_x H_n - \frac{1}{2} H^2 n_x \right) dL , \quad \mu = \chi + 1 , \quad (4)$$

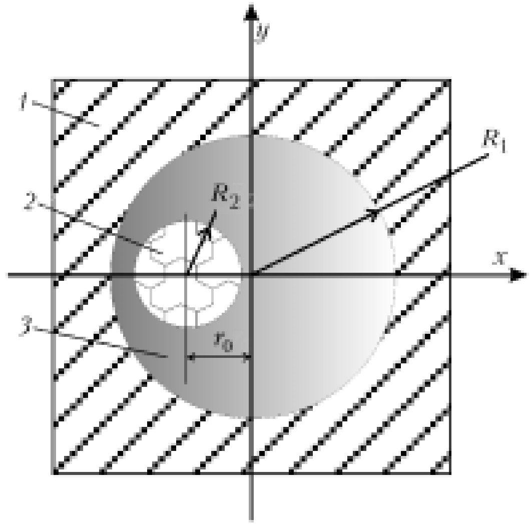


Fig. 1. Geometry of the problem: 1) bushing; 2) journal; 3) magnetic fluid.

$$\sigma_n = \mu_0 \left(\mathbf{H} B_n - \frac{1}{2} H^2 \mathbf{n} \right), \quad p = \mu_0 \chi \int_0^H H dH = \frac{1}{2} \mu_0 \chi H^2 , \tag{5}$$

where **n** is the vector of the normal to the contour and **i** is the unit vector of the x axis.

From the magnetostatic equations, it follows that the field intensity in all three regions is expressed by the magnetic potential $\mathbf{H} = -\nabla\varphi$ whose distribution is described by the Laplace equation

$$\nabla^2 \varphi = 0 . \tag{6}$$

On the journal and bushing surfaces, the conditions of continuity of the normal components of magnetic induction $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}_f)$ and the tangential components of field strength are observed:

$$B_{n1} = B_{n2} , \quad H_{\tau 1} = H_{\tau 2} . \tag{7}$$

The solution of problem (4)–(7) for the geometry presented in Fig. 1 is conveniently sought in a bipolar coordinate system [6]. The Cartesian coordinates are related to the bipolar coordinates α, β by the formulas

$$x = \frac{R_2 \sinh \alpha_2 \sinh \alpha}{\cosh \alpha + \cos \beta} , \quad y = \frac{R_2 \sinh \alpha_2 \sin \beta}{\cosh \alpha + \cos \beta} . \tag{8}$$

Let us assume that the coordinate surfaces $\alpha = \alpha_1$ and $\alpha = \alpha_2$ coincide with the surfaces of the bushing and the journal respectively. Bipolar representation of the field makes it possible to express the force (4) in the form

$$F_m = \frac{F_x}{F_*} = \frac{\chi (\chi + 1)}{(2 + \chi)^3} f_m , \tag{9}$$

$$f_m = \sinh \alpha_2 \sum_{k=0}^{\infty} \left(2k^2 d_k c_k - k (k + 1) (c_k d_{k+1} + d_k c_{k+1}) \right).$$

Here $F_* = 4\pi\mu_0 M_f^2 R_2 l$ is the quantity selected as the characteristic force scale;

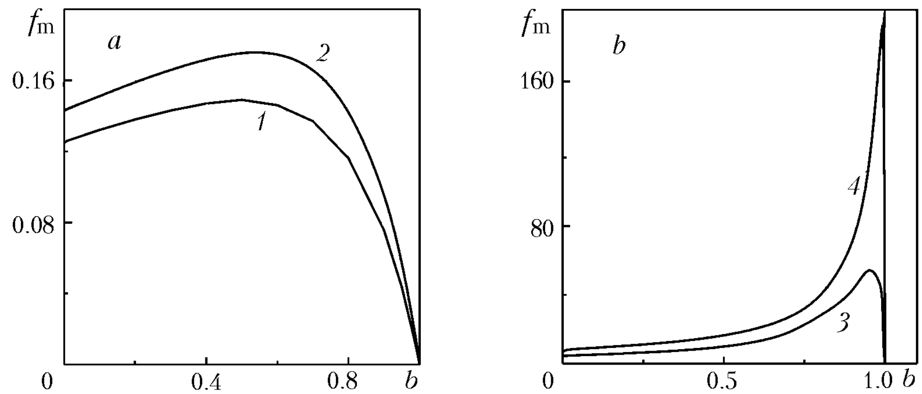


Fig. 2. Function of the maximum strength f_m vs. ratio b of the radii of the journal and the bushing for low (a) and high (b) values of the magnetic susceptibility of the fluid: 1) χ = 0.1; 2) 1; 3) 100; 4) 200.

$$d_k = \frac{\exp(-2k(\alpha_2 - \alpha_1))}{1 - \left(\frac{\chi}{\chi+2}\right)^2 \exp(-2k(\alpha_2 - \alpha_1))};$$

$$c_k = \frac{\exp(-2k\alpha_2)}{1 - \left(\frac{\chi}{\chi+2}\right)^2 \exp(-2k(\alpha_2 - \alpha_1))}; \tag{10}$$

$$\cosh \alpha_1 = \frac{1 - b^2 + a^2}{2a}; \quad \cosh \alpha_2 = \frac{1 - b^2 - a^2}{2ab};$$

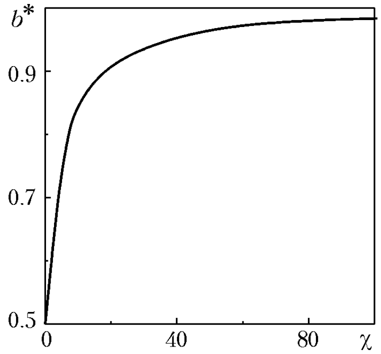
$$b = \frac{R_2}{R_1}, \quad 0 < b \le 1; \quad a = \frac{r_0}{R_1}, \quad 0 < a \le 1 - b.$$

According to (9)–(10), the dimensionless force is a function of the dimensionless parameters: the magnetic susceptibility of the field χ and the radius of the journal b and the displacement a of its center from the center of the bushing, which have been made dimensionless by means of the radius of the bushing.

A numerical analysis of problem (9)–(10) has shown that certain significant features of the dependence of the force on its arguments in [6] remain to be revealed. It has been established that the maximum value of the force F_m attained for the limiting displacement of the journal $a = 1 - b$ in the case of spherical contact [9] has an absolute maximum F_m^* (optimum force in what follows), when the relation b^* of the radii is optimum [11].

Figure 2 gives the dependences of the function f_m on the ratio b for the maximum displacement of the journal, which illustrate the properties of the force F_m. These dependences, as would be expected, have a maximum whose characteristics f_m^* and b^* are dependent on the susceptibility of the fluid χ. When the values of the susceptibility are low ($\chi < 0.1$), the optimum ratio is $b^* \to 0.5$ and f_m^* = 0.149. The dimensionless force f_m^* increases indefinitely with indefinite increase in χ; here, $b^* \to 1$. Thus, as χ varies in a wide range, the optimum values of the parameter b lie within $0.5 < b^* < 1$. The character of the dependence $b^*(\chi)$ is shown in Fig. 3. We note that the initial and final portions of the $b^*(\chi)$ curve have asymptotes.

Investigation of the extremum properties of the force according to model (9)–(10) is nontrivial. When the cylinders are in contact, the formulas of the bipolar coordinates lose their meaning; therefore, we must find f_m from the asymptotic dependence of the force on the displacement. Furthermore, by virtue of the mathematical peculiarity, the number of terms making a substantial contribution to the sum (9), as the journal approaches the bushing, indefinitely



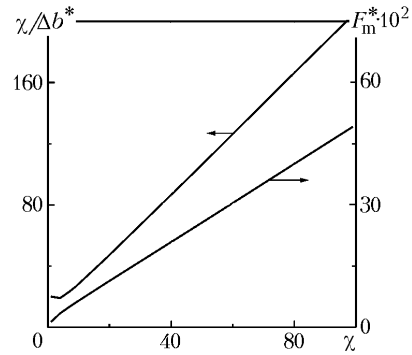


Fig. 3. Optimum ratio of the radii b^* vs. magnetic susceptibility χ.

Fig. 4. Parameter $\chi/\Delta b^*$ and optimum value of the force F_m^* vs. magnetic susceptibility χ.

increases. Therefore, construction of simple interpolation formulas reflecting the dependence of the force on its arguments is an important objective of study of model (9)–(10).

Interpolation Model for Calculation of the Limit of the Carrying Capacity of a Cylindrical Magnetofluid Support. For the sake of convenience, we represent the approximation of the dependence $b^*(\chi)$ (Fig. 3) in the $\frac{\chi}{\Delta b^*}(\chi)$ coordinates (Fig. 4), where it has an asymptote. For high values of χ, we have the dependence

$$b^* = \frac{\chi + 1.58}{\chi + 3.16}, \quad \chi \ge 20 . \tag{11}$$

In the region $\chi \le 20$, we may propose the formula

$$\frac{b^* - 0.5}{b_c} = \frac{\nu - 1 - \frac{\chi}{\chi_c} + \sqrt{\left(\nu + 1 + \frac{\chi}{\chi_c}\right)^2 - 4\nu}}{2\nu} \tag{12}$$

with the parameters χ_c = 0.79, ν = −10.27, and b_c = 0.45. We can replace Eq. (12) on the portion $0 < \chi < 1$ by the simpler equation

$$b^* = 0.5\,(1 + 0.1\chi) . \tag{13}$$

Table 1 compares the results of calculation of $b^*(\chi)$ from (11) and (12) to the results of model (9)–(10). The maximum error in using Eqs. (11) and (12) in the specified ranges of χ is no higher than 2%.

The dependence of the optimum force on the susceptibility, which is presented in Fig. 4 and has been calculated from (9) and (10), shows that, for high χ values, the function $F_m^*(\chi)$ has an asymptote described by the formula

$$F_m^* = (\chi + 3.7)/209 \tag{14}$$

on the portion $\chi > 10$.

The error of computations of ε_F from (14) in the above range of χ is no higher than 1.5% as compared to (9)–(10) (see Table 1, (14)).

For the portion $\chi < 10$ we may propose an interpolation formula describing the calculated data (9)–(10) accurate to 5%:

TABLE 1. Approximation of the Dependences $b^*(\chi)$ and $F_m^*(\chi)$ by Different Formulas

χ	b^*			$F_m^*(\chi)\cdot 10^2$			ε_F, %	
	(9)—(10)	(11)	(12)	(9)—(10)	(14)	(15)	(14)	(15)
0.1	0.501	0.515	0.505	0.177	1.818	0.176	927	0.5
0.2	0.505	—	0.510	0.338	—	0.331	—	2
0.3	0.507	—	0.515	0.481	—	0.471	—	2
0.4	0.510	—	0.520	0.626	—	0.600	—	4
0.5	0.517	—	0.525	0.755	—	0.722	—	4.4
0.6	0.520	—	0.530	0.878	—	0.836	—	4.8
0.7	0.530	—	0.535	0.993	—	0.945	—	4.8
0.8	0.535	—	0.540	1.103	—	1.049	—	4.9
0.9	0.545	—	0.545	1.208	—	1.149	—	4.9
1.0	0.55	0.62	0.55	1.308	2.249	1.246	71.9	4.7
4	0.71	0.78	0.69	3.408	3.684	3.405	8.1	0.09
7	0.80	0.84	0.80	5.000	5.120	5.033	2.4	0.70
9	0.84	0.87	0.85	6.004	6.077	6.000	1.2	0.07
19	0.92	0.93	0.92	10.856	10.861	10.208	0.05	6
49	0.97	0.97	0.94	25.231	25.215	20.820	0.06	17.5
79	0.98	0.981	0.945	39.596	39.569	30.551	0.07	22.8
100	0.984	0.985	0.946	49.165	49.139	36.852	0.05	25
200	0.992	0.992	0.948	97.476	97.464	67.840	0.01	30

$$\frac{F_m^*}{F_c} = \frac{2\chi\nu}{\nu - 1 - \dfrac{\chi}{\chi_c} + \sqrt{\left(\nu + 1 + \dfrac{\chi}{\chi_c}\right)^2 - 4\nu}} \cdot 10^{-2}, \tag{15}$$

where $\nu = 1.185$, $\chi_c = 32.3$, and $F = 0.298$ (see Table 1, (15)).

It is problematic to obtain the asymptotics by exact solution of (9)–(10) for $\chi \to 0$. From the viewpoint of the difficulty of accurate calculation, the magnetic force may be divided into two components: the "external" force calculated from the field of the magnet (noninductive approximation) and the "self-action" force due to the field induced by the fluid. However, in the case $\chi << 1$ the calculation of the magnetic force in the noninductive approximation is justified [2]. In this approximation, the magnetofluid pressure in (3) is only expressed by the distribution of the magnet's field; the second term (magnetic pressure jump on the body's surface) may be disregarded, since these assumptions introduce errors of the order of χ^2.

In the noninductive approximation, the force acting on the journal for low values of the magnetic susceptibility $\chi \to 0$ is equal to

$$F_m = \frac{\chi}{8} \cdot f_m, \quad f_m = \frac{(1-b)}{(2-b)^3}. \tag{16}$$

From the extremum condition $\dfrac{df_m}{db} = -\dfrac{1-2b}{(2-b)^4} = 0$, we find the optimum ratio of the radii $b^* = 0.5$ and the optimum force acting on the magnet, $f_m^* = 0.148$ and $F_m^* = 1.85\cdot\chi\cdot 10^{-2}$.

Calculation of f_m from formula (16) for $\chi \le 0.1$ yields the coincidence with the result obtained from formulas (9)–(10) in the entire range $0 < b < 1$. The computational error increases with parameter χ. When $\chi = 1$, the disagreement between f_m^* values computed from formulas (9)–(10) and (16) attains 16%.

The calculations carried out make it possible to obtain a theoretical limit for the carrying capacity of magnetofluid supports. From formulas (9)–(10), we have

$$\bar{p} = \frac{F_x}{2R_2 l} = \frac{F_* F_{\mathrm{m}}^*}{2R_2 l} = 5 \cdot 10^6 \, (\mu_0 M_{\mathrm{f}})^2 \, F_{\mathrm{m}}^*$$

for the reduced force (referred to unit area of the longitudinal section of the journal).

The real value for the induction of the magnet is $\mu_0 M_{\mathrm{f}} \approx 0.8$ T. The optimum dimensionless force F_{m}^* is determined from formulas (14)–(16). Since we have $\chi < 4$ for the majority of existing magnetic fluids, the theoretical limit for the reduced force will be 0.11 MPa ($\chi = 4$). We note that this value is technically unattainable at present because of the absence of magnetic fluids with a magnetic susceptibility of $\chi \sim 4$, which possess the necessary colloidal stability for $B \approx 0.8$ T. Commercial magnetic fluids have $\chi \sim 1$–2, as a rule, which makes it possible to ensure the carrying capacity of a cylindrical magnetofluid support at a level of 0.04–0.07 MPa.

Conclusions. A fundamental result of the numerical investigation of the analytical solution of the problem on calculation of the force of magnetic levitation of a cylindrical magnet in a nonmagnetic cylindrical bushing filled with the linearly magnetizable fluid is the existence of the optimum ratio of the radii of the bodies for which this force attains its absolute maximum (optimum force). The characteristics of the maximum are dependent on the susceptibility of the fluid. The optimum force indefinitely increases with indefinite increase in the susceptibility; the optimum ratio of the radii of the bodies varies from 0.5 to 1. This is of importance in technology for creation of supports, suspensions, vibration dampers, and dampers. However, the mathematical complexity of the formula makes it problematic to practically implement it even with the use of a computer.

The interpolation formulas proposed describe the dependence of the characteristics of the maximum on the susceptibility of the fluid with an accuracy sufficient for technical calculations and substantially simplify the procedure of calculation of the limit of the carrying capacity of cylindrical magnetofluid supports.

NOTATION

B, magnetic induction, T; F, magnetic buoyancy force, N; g, free-fall acceleration, m/sec^2; H, magnetic-field intensity, kA/m; H_0, magnetic-field intensity at the point (x_0, y_0, z_0); l, journal length; M, magnetization of the material, kA/m; M_{f}, magnetization of the magnet, kA/m; p, pressure, Pa; p_0, pressure at the point (x_0, y_0, z_0), Pa; r_0, displacement of the journal axis from the bushing axis, m; R_1, radius of the cylindrical bushing, m; R_2, radius of the cylindrical journal, m; x, y, z, Cartesian coordinates; μ, magnetic permeability of the material; μ_0, magnetic permeability of the vacuum; ρ, density of the magnetic fluid, kg/m^3; χ, magnetic susceptibility of the material. Subscripts: f, ferromagnetic; s, saturation state; h, halved magnetization of the material; *, characteristic value; m, maximum value. Superscript: *, optimum value.

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